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Lifetime Expected Income Breakeven Comparison Between SPIAs and Managed Portfolios

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ABSTRACT

- How do the total expected lifetime cash flows compare between Single Premium Immediate Annuities (SPIAs) and managing the portfolio in the markets?
- Use of Period Life Tables provides a better definition of the distribution period, a sense for probability of outliving certain ages, and recognition that the distribution period's length is a rolling time period rather than to a set, fixed age.
- Assets should be retained and managed in the markets, before the purchase of a SPIA should be considered, until older ages than currently thought.
- Key factors in order of impact on decision: Age (SPIA appropriate for older retirees), Longevity (higher likelihood of outliving peers), Allocation (breakeven depends on age & longevity expectations), Adviser Fees (least impact). All these factors should be considered in unison.
- SPIAs should not be considered for approximately two-thirds of the general population, and only at older ages for the rest.
- Research in the first part of this paper uses the "general population" Social Security tables. How does using a "healthier population" table like the 2000 Annuity Table affect the breakeven choice?
- Tables that have a shorter expected longevity period will favor managed portfolios while tables that have a longer expected longevity period will favor SPIA solutions.
- Longevity tables are different statistical subsets of the same overall population.
- The Annual Payout Rate (APR) that insurance companies calculate is a useful metric to simplify calculation of the breakeven point between retaining management of the assets and a SPIA.
- As interest rates increase, the APR may also increase. A practical application is described so that this comparative process may be simplified, and a comparison may be made in the future as insurance company Annual Payout Rates rise or fall.
- As the retiree ages, conditions may suggest switching to a SPIA (e.g., continued good health/longer expected longevity than cohorts); however, the APR comparison may be helpful for this determination.

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Brief Overview

Retirees have a choice to manage their retirement assets, or to annuitize some, or all, of those assets. Annuities (SPIA) provide protection against market fluctuations and protection against longevity, though generally no protection against inflation and no end-of-life bequest. Managed portfolios provide less certainty of return but more opportunity for growth and for the potential to leave a legacy.

Questions this paper seeks to answer are: What are breakeven asset allocations below which a SPIA provides a higher lifetime expected total cash flow? At what age(s) does it make sense to start with a managed portfolio, and when to switch to a SPIA? Managed portfolios retain a balance at death while SPIAs have none. How does the cash flow breakeven comparison change when that balance is, or is not, considered? Does age matter in the decision to switch from a managed portfolio to a SPIA? Do conclusions depend on the choice of mortality tables used in the analysis (Social Security Table, "General Population" vs. Annuity 2000 Table, "Healthy Population")? How do market returns affect the breakeven comparison? How do fees affect the comparison?

This research looks at the decision from a comparison of the sum of total annual retirement income payments received. Implicit in this comparison are three assumptions; 1) that retirees value consumption equally in all years of retirement, 2) that the sum of lifetime income is an appropriate proxy for the present value, and 3) that retirees do not value bequests equally. Regarding bequests, some retirees may be concerned about either having sufficient funds to fund further income beyond expected longevity or about leaving a bequest of uncertain value, while other retirees have no bequest desire and wish to consume their assets as their own income. These two desires lead to the conclusion that it is equally important to have adequate retirement income in each individual year of one's life for as long as that may last, but may not be as important to retain an ending portfolio value for whatever reason (or vice versa). These competing desires define the special case which serves as a base upon which retirees can value their own preferences.

The Monte Carlo model used tracks cash flow and portfolio balances by age and allocation and determines the effect of prior cash flows on future balances. By comparing the total sum of payments received from a SPIA against the payments expected from a managed portfolio, retirees may determine which approach might maximize their lifetime consumption. Retirees can also predict the age when the cumulative cash flows from a managed portfolio will exceed those from a SPIA and the likelihood of outliving this breakeven age based on current withdrawal rate management technology and currently available data.

The future, by definition, is unknown. This study helps retirees determine if the simplification of the longevity question and the presumed reduction in market risk from purchasing a SPIA is worth the price in reduced lifetime consumption. This is part one of a two part paper with more data available in a working paper posted online.[1]

Brief Literature Review

Blanchett (2013) stated “Cash Flows are the most important determinant of retirement success.” Yet, past distribution research has not focused on cash flows, but on maximizing the withdrawal rate, which Blanchett explains is more a function of the returns data used in research papers. This paper’s focus will be on cash flows that emanate throughout a simulated retirees' potential lifetime as developed by the methodology in Frank, Mitchell, and Blanchett (herein FMB) 2011, 2012a and 2012b. The FMB methodology is Age-Based where annual cash flows may be determined and summed over various expected longevity percentiles from Period Life Tables.

Milevsky (1998) stated "Consequently, given the low interest rate environment and the long-run propensity for equities to outperform fixed income investments, … a sixty-five year old female (male) has a ninety-percent (eighty-five percent) chance of being able to beat the rate of return from a life annuity until age 80." That was his conclusion with interest rates in Canada in 1998. Do those observations hold with lower, or higher, interest rates? Do they hold for other retiree ages older than age 65 (i.e., if a retiree waits, at what age should they reconsider a SPIA)?

Economists refer to the "annuity puzzle" as research since Yaari (1965) questioned why annuitization is such an uncommon solution for protecting from longevity risk. Numerous explanations have been offered and Benartzi, Previtero, and Thaler (2011) effectively summarize this literature. More recently, Reichling and Smetters (2013) indicate that because health shocks require increased liquidity while simultaneously reducing the present value of the remaining annuity payments (to zero at death) as a result of heightened mortality, retirees act rationally in their decision to reduce annuity holdings. Kitces and Pfau (2013) also demonstrated that part of the explanation for the beneficial impacts of annuitization on a retirement portfolio can be explained through the implied rising equity glide path of the strategy.

With regard to research which compares the cash flow provided by different retirement income strategies, Pfau (2011) and Pfau (2013) both analyzed the income stream supported by income guarantees on variable annuities to see how likely it could be for a systematic withdrawal strategy to replicate these payments. This literature review is meant to cover the main highlights on the topic explored by this paper and is not all inclusive.

A Probability-Based Three Dimensional (3D) Model

Fundamentally, a retiree has three basic choices about how to fund their retirement income. They can annuitize their retirement assets, they can manage those assets individually or through an adviser, or partially annuitize and manage the balance. At the end of the day, what is the total sum of money a retiree may expect, under either choice, over their remaining expected lifetime? How does safety based (SPIA) total payouts compare to probability based (Monte Carlo) total payouts over the same time period established by the period life tables?

At any moment the future is unknown. However, Monte Carlo simulations may be used to provide insight into that unknown future. Insurance companies establish SPIA payouts based on what is known today as well – yet the future is equally unknown to all participants.

The Monte Carlo simulations for this project are run using a three dimensional model developed by Frank, Blanchett, and Mitchell, and described in FMB 2011, 2012a and 2012b. This paper is a practical application of the FMB model to compare total cash flow sums between SPIA and managed portfolios by allocation exposure to equity and bonds, and by expected longevity percentile (LP) (probability of outliving a particular age) from period life tables.

The FMB approach is not a 4% rule approach. In the FMB 3D model, for the retiree's current age, period life tables are used to set the length of the distribution period *each year* for retiree ages from age 65 and up; thus the length of the distribution period (DP) *changes each year* throughout the retiree's lifetime based on their current cohort age and the respective expected longevity age. By being more specific as to the length of the DP through the use of Period Life Table Longevity Percentiles (LP), and rolling simulations that are serially connected through an annual update, it is possible to get more visibility on how cash flows change throughout retirement.

A constant probability of failure (POF), which is the percentage of simulations that fail, is also set to determine the specific withdrawal rate (WR) based on the DP described in the earlier paragraph. As described in FMB 2011, it is the interaction between the length of the DP and the POF that sets a specific WR for each retiree's current age. Asset allocations between 0% equity (100% bonds) through 100% equity (0% bonds) add a third dimension. Thus, the WR, given a target POF, is annually recalculated based on each annual DP and asset allocation. This paper sets the Monte Carlo simulation target POF for all values at 10%. However, because the WR is annually recalculated, the effective POF is essentially zero since there is always a portfolio balance; the annual cash flow would be unknown unless wisely managed. In other words, there is a tradeoff between spending and portfolio balances.

An important difference between the "4% rule" approach and a 3D model approach is that the length of the DP, *combined* with a target POF, determines a given withdrawal rate (WR) that holds for that single particular year, *regardless of portfolio value*. The withdrawn dollars each year are mathematically connected to the supporting portfolio value *each year*. The DP is specifically determined by reference to a Period Life Table each year and the DP itself may be changed through strategic use of Longevity Percentiles within Period Life Tables. In other words, the WR is determined in the 3D model by the length of the DP at a given POF *and* a given LP. Possible subsequent annual withdrawal dollars (annual cash flows) and resulting annual portfolio values result from the WR calculation in this 3D model, not the reverse, and it is this annually recalculated, rolling DP/POF-determined-WR that is subsequently applied to the portfolio balance.

A result of the difference in model design is that there is always a portfolio balance to which a WR may be applied because WR never reaches 100% (or unless simulation portfolio values go negative) through a limiting function to WR described in FMB 2012b (WR*(1 – 1/n) where n = annually recalculated DP).

As in real life, the WR is recalculated each year and cash flow results are serially connected by portfolio value calculations post cash flow reduction. Past actions such as withdrawing more, or less in any given year, affect portfolio values the retiree has today; and the present portfolio value of the retiree affects what sustainable withdrawal dollar amounts are possible for the coming year(s). The effect of future market sequences on portfolio values are not directly under the retiree's control beyond asset allocation, and allocation is a component of the 3D model to determine its impact (FMB 2011).

The WR increases slightly each year because the DP shortens due to a slightly shorter expected longevity from the period life tables. This simulates the retiree aging and the dynamics of changing life expectancy as they age. The DP continually shortens as the retiree ages according to the updated expected longevity age for the retiree's current age (DP = $\text{Age}_{\text{Expected Longevity}}$ minus $\text{Age}_{\text{Current}}$). Expected Longevity uses the 50^{th} longevity percentile (LP) where 50% of cohorts outlive the DP and Extended Longevity uses the 30th LP where 30% of cohorts outlive the DP. Table 1 summarizes corresponding DPs.

With an established WR based on age, as described above, the actual dollar amount for a retiree at each age depends on how the markets have affected the retiree's portfolio value the prior year. Annual Portfolio Value multiplied by the WR equals the Annual Dollar Distribution.The annual portfolio value, and thus annual dollar distribution is based on "good" or "bad" simulated markets. The 25th (best one in four) percentiles represent the "good" market sequence cash flows and the 75th (worst one of four) percentiles represent the "bad" market sequence cash flows. The 50th percentile represents the median sequences.

Table 1. Distribution Period Lengths where X% of cohorts outlive determined age.

Retiree's Current Age	Expected Years (50% Outlive)	Expected Age at Death (50% Outlive)	Extended Years (30% Outlive)	Extended Age at Death (30% Outlive)
65	21	86	26	91
70	17	87	21	91
75	13	88	17	92
80	10	90	13	93
85	7	92	9	94

Social Security Tables (2007)

Finally, since managed portfolios usually have management fees, results will compare sensitivity at no fee, one-half percent fee, and one percent fees. Investor adviser fees and investor taxes would be applied to the cash flow that results from the WR applied to the portfolio balance. In other words, the annual cash flow is gross of fees and taxes. Since taxes may vary greatly, the values computed here are pre-tax, but post a range of adviser fees. Fees are considered in the managed portfolio for an apples-to-apples comparison, since the SPIA dollar amounts are net of SPIA fees to the retiree. Briefly, the reader will see that, although fees have an effect, they are not as large as other factors investigated (see Figures 1 and 2).

It will be the sum of these annual cash flows, based on simulation percentile, which this project will compare to the sum of the SPIA payments, over the identical expected longevity from the period life tables. These sums will also be similarly determined across the third dimension of asset allocation.

It should be noted that the market data and figures are as of year-end 2012 and SPIA data is from 2013 reflecting how a real life retiree would compare a SPIA with a managed portfolio. These will in all likelihood change over time in the future as market returns, SPIA rates, and longevity data (Life Tables) are updated since all of these are stochastic in nature. Client age and portfolio values would also be a part of the annual update process in a dynamic model reflecting actual data and spending events in life.

In sum, what is unique to this paper is the methodology to track cash flows annually through a portfolio distribution's lifetime, by simulation percentile and asset allocation, and compare that cash flow to that of a SPIA.

Methodology – Comparison between Annuitization and Monte Carlo Simulation

Returns are based on four main asset classes from Ibbotson® SBBI® through December 2012: 30 day T-bill, US Intermediate-term Government Bonds, S&P 500 Stock, and US Small Stock. All returns are converted into "real returns" (i.e., adjusted for inflation) using SBBI US Inflation.

Social Security (2007) Period Life Table (available at http://www.ssa.gov/OACT/STATS/table4c6.html).

2000 Annuity Life Table (available at http://www.soa.org/ (search term 2000 Annuity Table).

Present SPIA payouts are used from http://www.immediateannuities.com/ and discounted over the stated period by the same inflation from the Ibbotson data above.

Methodology compares an expected total sum of money received between a SPIA and managed assets, over a given time period, in real terms, across various asset allocations (from 0% equity to 100% equity). Past FMB study methodology employed annually recalculated simulation results, based on asset allocation, *rolling* portfolio values and withdrawn dollar amounts *each year*,; serially connected to each other as they would be in real life. New to this study are fees at the 0%, 0.5% and 1.0% levels calculated to determine a range of fee sensitivity. The depicted sum of distribution dollars are between the 25th and 75th simulation percentiles to illustrate the most likely boundaries that a retiree may experience *throughout* the remainder of their lifetime (i.e., portfolio values would most likely fluctuate within this range of values).

As described in the overview above, a retiree today faces a choice between annuitization and management of retirement income. Everything about the future, known today under either choice, is equally as blurry. How do the choices compare to a consumer in terms that matter to them?

First, what is the present monthly SPIA payment available for Mary, a 65 year old female? Subsequent values to be annuitized at later ages come from the 50th percentile portfolio balance at each future age (using 40% equity value). This is because, at age 70 for example, Mary may not have annuitized, but managed her monthly income, through the model described earlier until reaching a later age. All portfolios are managed with withdrawal adjustments based on age as described in FMF 2012b in order to extend portfolio values into any superannuated age that a retiree may reach.

The starting value is $1,000,000. From immediateannuities.com for Mary, a 65 year-old female in Texas (a State with no SPIA premium tax; thus values here are pre-premium tax[2]) today would receive a Single Life with No Payments to Beneficiaries monthly payment of $5,170. Compare this SPIA payment to the monthly payment of $4,260 per month from the 3D model at age 65.[2]

It initially may appear that the SPIA will generate more income than a managed account using these beginning values. However, the SPIA is fixed and the cash flow from the managed account is based on the markets' inflation adjusted real returns. To make the numbers comparable, the SPIA is adjusted for inflation (average 3.0401% over the study period). Based on the expected remaining lifespan (N) of 21 years (Table 1) the inflation-adjusted present value for the SPIA = $\sum_1^N$(monthly SPIA*12)/((1+0.030401)^N), is $952,664. The managed account is expected to generate $1,633,412 (40% equity, 1% fee, including portfolio balance at end of 21 years) or $1,125,936 (excluding portfolio balance after 21 years).[2] Mary should expect an extra $173,272 (1,125,926-952,664) of purchasing power in current dollars over her lifetime with the managed account plus a residual balance of $507,476 that may be used for bequest or continuing income.

Figure 1 summarizes the expected total income sums at various allocations, simulation percentiles (25th good sequences, 50th median, and 75th poor sequences), fees, by various ages that are time slices through the 3D model. In reality, market return sequences would tend to fluctuate around the median values. Monte Carlo simulations do not predict the future, but they do provide a range of possible outcomes upon which decisions may be made. The left side of Figure 1 uses expected longevity while the right side uses extended longevity ages and resulting DPs (Table 1). Figure 2's only difference is that ending portfolio balances are not included in the total lifetime cash flows to equalize the comparison is a retiree gives up their portfolio to buy a SPIA.

Rather than an assumed zero portfolio value at the end of any given distribution period, this paper looks at the range of possible portfolio values at the simulation's end age, which represent future income potential either through a managed portfolio approach, or through purchase of a SPIA at that end age, for income to continue beyond that age because the retiree is still alive. Therefore, this is a timeline approach where the decision to get a SPIA is delayed and comparison made going forward from those later ages so that market return sequence risk may be better evaluated from those changing ages going forward.

The methodology provides a deeper insight into how long a retiree needs to outlive cohorts by comparing expected longevity (50th percentile as shown in longevity tables) to the 30th percentile where a retiree outlives 70% of their cohorts. Further research in needed to provide more insight regarding the sensitivity of the breakeven between SPIA lifetime total cash flow compared to that of a managed portfolio as a function of longevity percentile.

Figure 1. Breakeven Comparison Expected Lifetime Total Cash Flow (w/ Ending Balance)

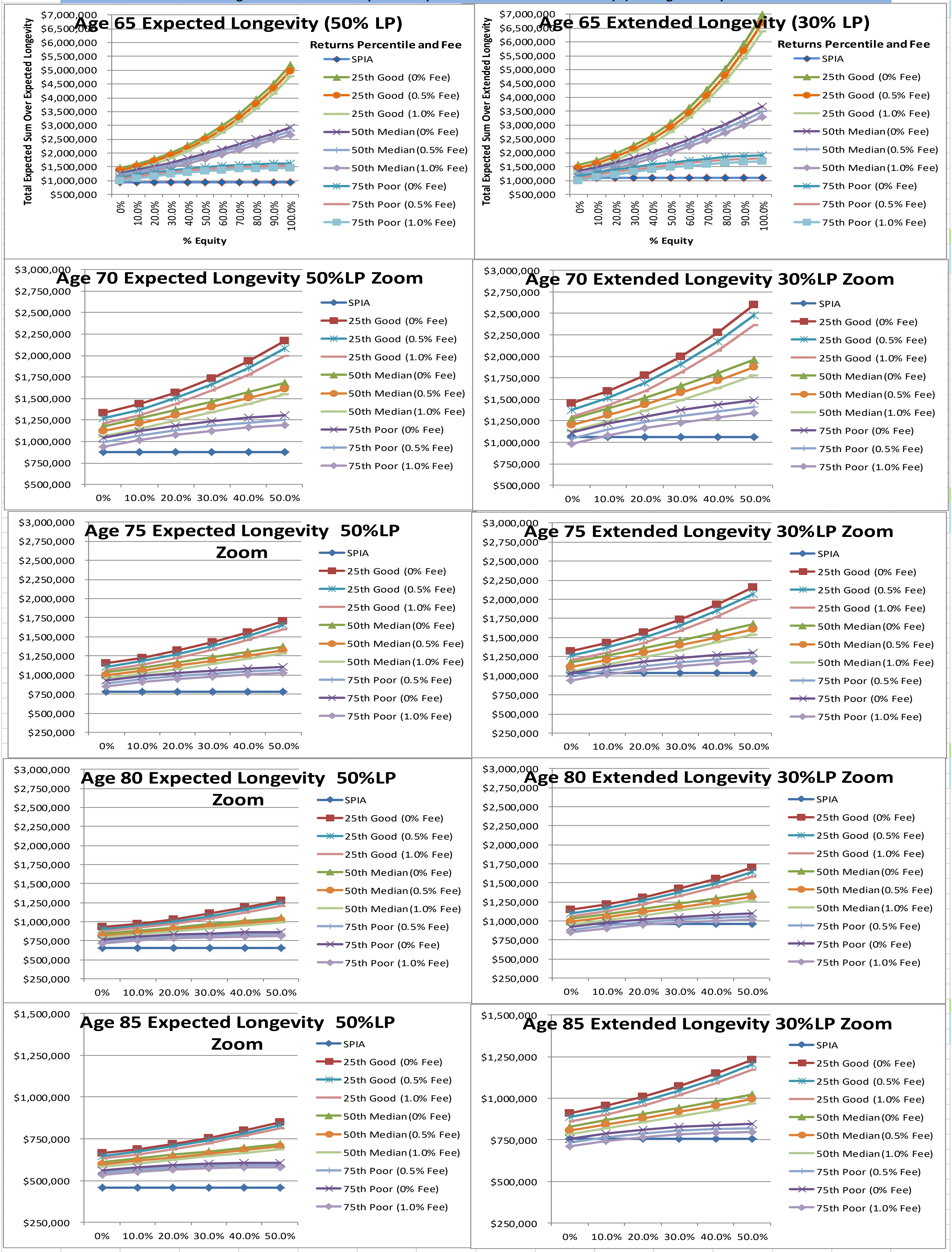
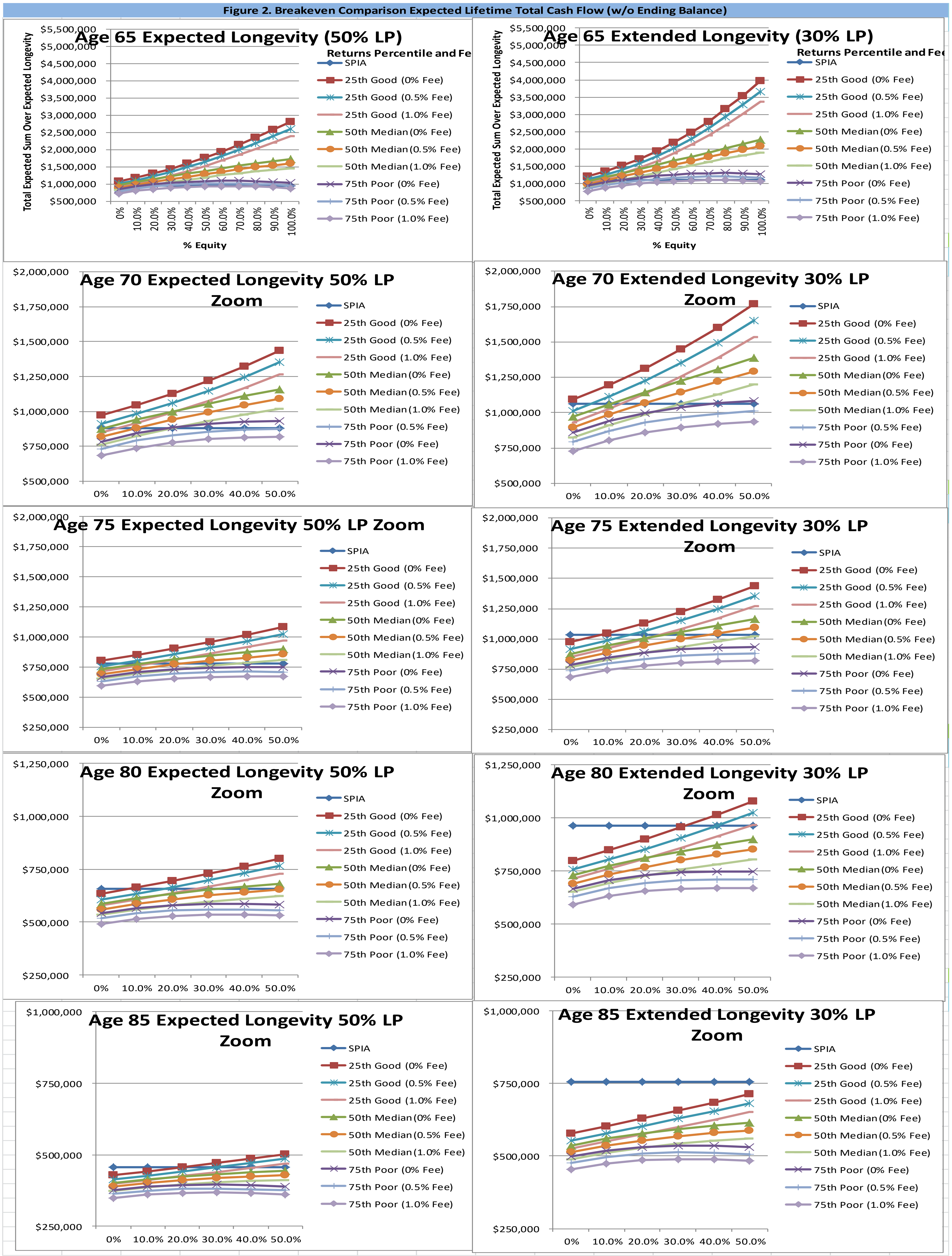


Figure 2. Breakeven Comparison Expected Lifetime Total Cash Flow (w/o Ending Balance)



Those graphed values in Figures 1 and 2 that lie above the "SPIA" line are above that breakeven value, while those graphed values that lie below the "SPIA" line are below that breakeven value. For example, if none of the managed portfolio total income sums fall below the annuity breakeven line, with or without ending expected ending portfolio values, a SPIA, in those situations, would be the less efficient manner to get better total lifetimes sums of income.

What a SPIA does is provide protection against living through extensively long poor markets, i.e., longevity during bad times. The reason that SPIA begin to make sense for older retirees is that it is easier both to have a significantly long bad market compared to expected lifespan at greater ages AND at greater ages the variability of remaining life increases relative to expected longevity (Mitchell, 2010). So both parts of the risk avoided by SPIA are increasing. However, what cannot be observed in the figures is the risk of SPIA failure (insurance company fails) due to low returns during those same long poor markets.

Figures 1 and 2 illustrates the dynamics of time and portfolio growth as well as the effect a higher withdrawal rate has on the remaining portfolio balance. You may think of the figures as seeking an age where crossover from a managed portfolio to a SPIA may occur. When ending portfolio balances are added back to the total cash flow *at expected longevity age* (assumes the retiree died as expected), in all cases the sum total of cash flow from the portfolio exceeds the breakeven line representing the cash flow from the SPIA.

In most cases, the longer the time a portfolio has for growth, the greater the difference between the total sum from a SPIA and a portfolio, in other words, a 65 year old has longer to reach their early 90's as compared to an 80 year old.

Panels in Figures 1 and 2 labeled "Zoom" focuses on the allocations at and below 50% equity since these are the allocations a retiree would more likely be comfortable with. Higher equity allocations have more cash flow because of the nature of higher returns associated with those allocations, however the increased volatility also associated with increased equity exposure is visualized by the wider dispersion of simulation percentile results between 25th and 75th Percentile results.

Portfolio values would most likely fluctuate much closer to the median value, especially if spending adjustments as described in FMB 2011 are used. Barberis (2013) describes the emotions of spending adjustments "The intuition is that, upon receiving a negative income shock, the individual prefers to lower *future* consumption rather than current consumption. After all, news that future consumption will be lower than expected is less painful than news that current consumption is lower than expected. Moreover, when, at some future time, the individual actually lowers consumption, the pain will be limited because, by that point, expectations will have adjusted downwards." Thus, if a retiree is pre-prepared for an eventuality ahead of time through discussing the possibility, either positive or negative spending patterns, when the time comes the pain may be more muted.

Managed portfolio cash flows that plot above the SPIA line represent total cash flows that exceed the SPIA cash flows for the DP in Table 1, and managed portfolio cash flows that plot below the SPIA line represent total cash flows that do not exceed the SPIA cash flows for the same time period.

The age shown in each figure is the retiree's current age. The results shown in each figure represent the values *after* the expected numbers of years from the life table have passed.

What the figures show that hasn't been shown in previous research, is the degree of risk (equity exposure) a retiree with a managed portfolio may be willing to take, based on their age as part of their decision to annuitize or not. If they are comfortable with an equity allocation higher than that indicated in a figure, then they should continue managing assets. If not comfortable, then they should annuitize. Notice too, and this is very important, that the SPIA only begins to come into play for those who have a tendency to outlive their cohorts and only at advanced ages when they no longer have a long remaining lifespan over which to generate market returns. And this is the real purpose of SPIAs; transferring the greater relative longevity risk at advanced ages to SPIA providers.

The insight here is that at any age, in order to justify self-managing their portfolio, a retiree does need to have both a sense of likely being in the group of long lived cohorts and a comfort level with the equity allocation required to do better than a SPIA. Part 2 to this paper will demonstrate how this comparison may be made in a spreadsheet.

Note that ending portfolio values are not affected by fees and taxes. The methodology used in this paper is to take fees (0.5% or 1.0%) from the annual cash flows (fee comparisons are depicted in the figures). In other words, fees are considered part of the sustainable annual withdrawal cash amount. Taxes would be treated in the same manner, by coming from the annual cash flows, and are not included in this paper since taxes are situation dependent, i.e., distributions are pre-tax in this paper. The effect then is that the net after-tax cash flow would be different for different retirees. However, the gross cash flow is not affected by taxes using this method, and is thus comparable.

Observations

If the retiree has a bequest goal, then Figure 1 shows that even for those who suspect they may outlive 70% of their cohorts (right side), let alone just expected longevity (50%), managed withdrawals plus retained value falls above the SPIA breakeven line. Other than in the most pessimistic of market expectations and risk aversion, managing the portfolio makes sense at all ages. The retained portfolio value still remains to provide income beyond the ending age at the end of the DP in Table 1 should the retiree still remain alive (FMB 2011, 2012a, 2012b).

The portfolios' ending value becomes irrelevant when a SPIA is purchased, and thus Figure 2 comes into use since both comparisons would not have an ending value (although, until the SPIA is actually purchased, the retiree does in fact still retain an ending balance, although uncertain as to value). In this case, the right side of Figure 2 suggests that the earliest a breakeven assessment should be made is possibly after age 70 (for the most market pessimistic of retiree, who is also most optimistic to outlive 70% of their peers). While the left side of Figure 2, for the average retiree who simply feels they'll meet expected longevity (50%) the age to review switching to a SPIA is around age 80.

By their nature, a SPIA requires a retiree to purchase, at some particular age while remaining lifetime is uncertain, because this is the mechanism where mortality credits originate. Mortality credits happen because those who die before the expected age fund those who continue to live longer than expected. Clearly someone in poor health should not use a SPIA since they most likely would not live long enough to recoup their money or mortality credits.

The current low interest rate environment, where SPIA cash flows are low as a result, suggest that policy decisions about encouraging the purchase of SPIAs prior to age 80 may be misplaced. However, a SPIA may be appropriate if the retiree may demonstrate a tendency as a spend thrift and a managed portfolio approach therefore may not be appropriate to ensure a prudent income for life. Buying a SPIA today may avoid the risk of lower SPIA rates later; or may lock in lower rates today relative to potential higher SPIA rates later.

This research demonstrates a breakeven evaluation process between self-managed portfolios and SPIAs, not a risk-adjusted value of either approach. The purpose of the paper is to demonstrate a straight forward method to determine breakeven points. Due to the stochastic nature of all the inputs, it is not a one-time-and-decide process. Instead, this process should be revisited each year and all relevant factors considered until such time the retiree makes the decision to switch from managing their assets to a SPIA.

If a retiree is concerned about *outliving* the ages in Table 1, *and not comfortable* with an allocation above the SPIA breakeven line in Figure 2, then they should consider purchasing a SPIA based on their current age. On the other hand, if they do *not* believe they'll outlive the ages in Table 1, then the benefits of a SPIA that go hand in hand with insuring income for long life would not be realized and thus the retiree should *not* buy a SPIA, especially if they *are* comfortable with an allocation *above* the SPIA breakeven lines. If they retain ownership of their portfolio by not buying a SPIA, then the relevant figure is Figure 1 since the ending balances are still retained by not buying a SPIA.

Not all risks are the same. If there is a risk of the retiree being a spendthrift, then a SPIA would provide income for life and avoid such risk. If there is *little* risk of long life, transferring assets to a SPIA to insure against that risk is not efficient. At what point does transferring assets to a SPIA make sense? The results above suggest that only when the possibility of outliving 70% or more of your cohorts exists, and then only at elderly ages, although the retiree will always have unknown remaining life expectancy. For ages younger than 75 to 80's, the assets are best kept within the family and future heirs since both inflation and possible future market returns have time to do better than SPIA lifetime sums do.

The methodology used in this paper for a SPIA may also be used to evaluate a decision between choosing a pension lump sum offer, which would then be managed in a portfolio, vs. accepting the pension payments for life (no COLA), which would be comparable to a SPIA. A factor, not considered in this paper, is that a retiree should consider the business risk of the insurance company; will they remain in business as long, or longer, than the retiree is alive? SPIA payments should not be considered completely risk free. Since the insurance company issuing a SPIA invests in the same markets as do the managed portfolios, if the retiree concern is continued poor markets (e.g., the 75th percentile managed values), then the risk increases that the insurance company cannot continue to afford a SPIA established during better times.

Further research is needed to look into possible combinations of partial annuitization over time schemes, and/or use of longevity annuities (deferred income annuities or DIAs) which don't payout unless a later age is actually reached. The purpose of this paper is to introduce the concept of cash flow comparisons.

Up to this point

The data up to this point in the paper has been derived from the general population Social Security table. How does choice of life longevity table affect the breakeven point? Also, is there a simple way to do these comparisons?

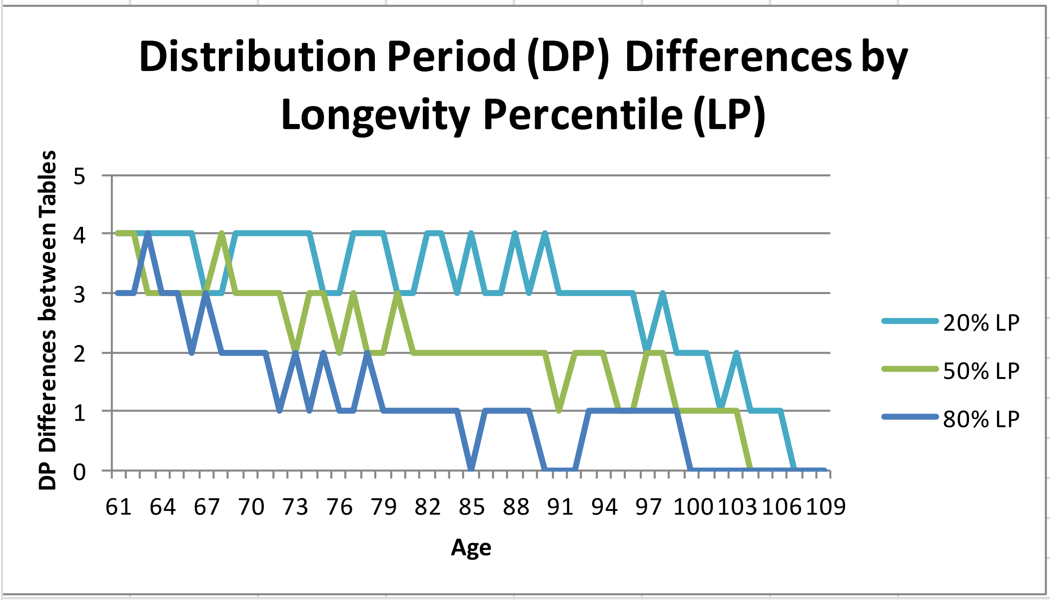
The following segment of the paper will cover a practical method to determine a relative breakeven comparison after showing the reader the differences between a healthy population (2000 Annuity Table) and general population (Social Security Table). At the end, the paper will also discuss the practical use of the SPIA Annual Payout Rate (APR) to determine breakeven thresholds.

Brief Overview for remaining half of paper

The paper above compared self-managed portfolios with the option of a SPIA using the Social Security "general population" tables.
What about using other Longevity Tables for a "healthier population" such as the 2000 Annuity Tables? How does using longer distribution periods that result from using such a table change the comparison?

Figure 3 shows the differences (Annuity Table minus Social Security Table) in Distribution Period (DP) lengths, by Longevity Percentile (LP) (percent of cohorts that outlive cohorts at any given age). At age 61 the Annuity Table results in an additional 4 years of DP at a LP of 80% or less. As a retiree ages, the difference between the tables becomes smaller. Retirees with low LP expectations (expect to live longer than average, e.g., 20% LP expect to outlive 80% of their cohorts) are affected more by the difference in table choice. Retirees with high LP expectations (e.g., 80% LP expect to outlive 20% of their cohorts) the table choice makes no more than a one-year difference after age 78. Notice that both tables begin to merge at older ages. Basically the effect, seen later, of a shorter (longer) DP is a higher (lower) WR and thus slight higher (lower) annual cash flow. More on the impact between tables may be found in FMB 2011 SSRN.

Figure 3. Distribution Period (DP) Differences: Social Security Table (Shorter DP, "General Population") and 2000 Annuity Table (Longer DP, "Healthier Population")



Source: FMB 2012 SSRN

The reader may compare Table 2 Distribution Periods to the Social Security DPs to Table 1 above.

Table 2. Female Example (Longer Longevity) Comparison between Annuity 2000 (Healthy Population) Table and Social Security (General Population) Table

Annuity Period @ 50% Outlive		Social Security Corresponding XX% Outlive* Annuity Period			Annuity Period @ 30% Outlive	
Age	# Years					# Years
60	29	>	38%	18%	<	34
65	24	>	39%	19%	<	29
70	20	>	38%	20%	<	24
75	15	>	42%	22%	<	19
80	11	>	39%	21%	<	15
85	8	>	37%	23%	<	11
90	6	>	39%	23%	<	8
95	4	>	43%	22%	<	6
100	3	>	46%	18%	<	5
*50% = Expected Longevity by definition				*30 -> Extended Longevity		
The center percentages (Social Security) represent the % of "General Population" cohorts that live as long, or longer, than their "Healthy Population" (Annuity) cohorts.						

The annuity table is a statistical subset of the overall general population. Table 2 compares the healthy population table (2000 Annuity) expected longevity periods with the percent of the general population (Social Security) table that outlive those same longer annuity-defined time periods. Some differences are due primarily to the rounding algorithm that determines table percentiles. The left half of Table 2 shows, at any given age, approximately 60% of the “general” (Social Security) population dies by the same time period (#Years) that corresponds to “healthy” (2000 Annuity) population expected longevity (50% outlive by definition in both cases). Similarly, the right half of Table 2 shows approximately 80% of the general population dies by the same time period (#Years) that corresponds to the healthy population’s 30% LP. A retiree really doesn’t know which percentile they are in except by continuing to live. Mitchell (2010) shows that, as retirees age, the uncertainty of their remaining lifespan (coefficient of variation) increases. All life tables are subsets, of the same population, that overlap. Thus, use of any table would not produce unexpected results over another, since any table could be used by adjusting longevity percentiles to achieve similar results of another.

Since the breakeven point, in the first part of this paper, began to favor SPIAs when the Social Security longevity percentile was where 30% of the cohorts outlived their peers, the comparison in this paper is demonstrated using that same longevity percentile. What is the long term effect of using a more conservative period life table (one that results in longer distribution periods)?

Figure 4. Breakeven Comparison Expected Lifetime Total Cash Flow (w/ Ending Balance) Social Security Left Side / 2000 Annuity Table Right Side

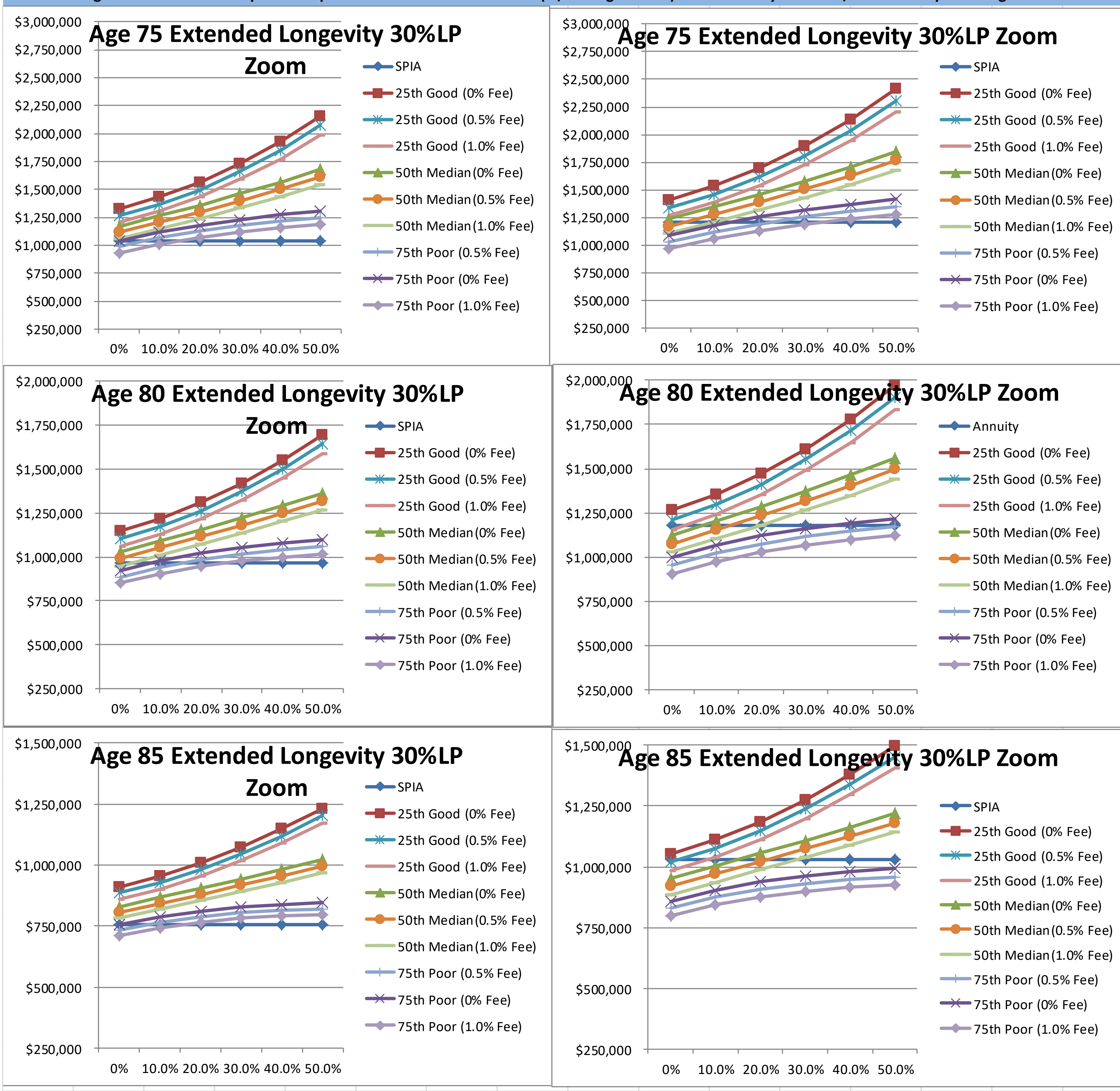
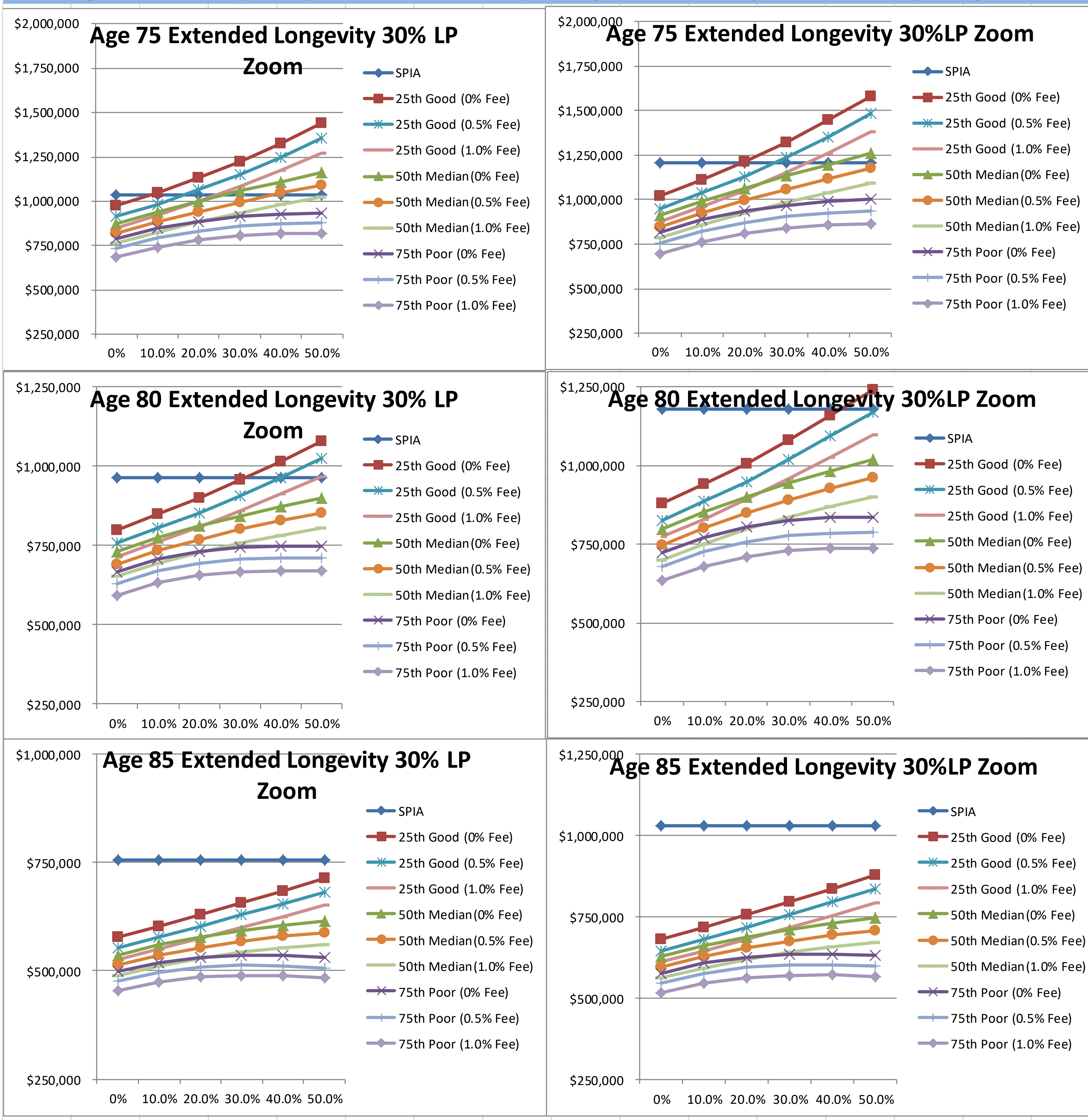


Figure 4 illustrates the breakeven sums for a retiree wishing to leave a bequest; therefore ending portfolio values are included in the total lifetime sums. Figure 5 is for a retiree without a bequest motive and thus the ending portfolio values are not included in the total lifetime sums of the managed portfolios. A retiree with a bequest motive would be psychologically harder to switch to a consumption goal by purchasing a SPIA. The retiree should evaluate their comfort level with how much exposure to equity and well as their continued likelihood of their being in the group outliving 70% or more of their cohort.

Figure 5. Breakeven Comparison Expected Lifetime Total Cash Flow (w/o Ending Balance) Social Security Left Side / 2000 Annuity Table Right Side



The first observable effect from lower relative withdrawal rates is on the portfolio balances where lower rate balances are higher relative to higher withdrawal rate balances at any given comparison age. How does this affect the total lifetime cash flow sum comparisons and breakeven point? A comparison of figures 4 and 5 with Figures 1 and 2 reveals the following observations:

- Lower overall withdrawal percentage rates lead to higher relative portfolio values at any given age (withdraw less = retain more).
    - For example: because of the greater preservation of portfolio values, the retiree has a greater asset value with which to purchase a SPIA at a later age. The higher portfolio value results in a higher SPIA monthly payment at older purchase ages. Example at age 85 using the Social Security Table, a median simulated portfolio value of $739,000 may purchase a monthly SPIA income of $8,100. Using the Annuity 2000 Table, the median simulated portfolio value of $830,000 may purchase a monthly SPIA income of $9,291.
- Because portfolio values are preserved due to lower withdrawal percentage rates using longer distribution periods from the Annuity table, the result is a higher lifetime total cash flow for the slightly longer distribution period (although each years' cash flow, using longer DPs, would be lower relative to that of shorter DPs; it is this effect that preserves portfolio balances for subsequent years).
- The higher monthly SPIA income raises the total remaining cash flow sum breakeven point. The managed portfolio has less time remaining for market returns to overcome the higher breakeven value. Thus, using period life tables that result in longer distribution periods (i.e., Annuity Table) yields lower withdrawal percentage rates that tend to preserve portfolio value for later SPIA purchase.
- These results are consistent with the bequest motive findings in FMB 2012b where lower percentage withdrawal rates that result from using longevity percentiles that fewer retiree cohorts may outlive, due to portfolio values preserved for bequest instead of using higher relative percentage withdrawal rates that would be consistent with a consumption motive.
- A retiree having a bequest motive may find it more beneficial to switch to a SPIA at a later age because the result of their motive would be higher relative portfolio values which may generate higher SPIA income. It turns out that a bequest motive and a concern about outliving assets are consistent and mutual concerns that may be achieved with income from a properly measured portfolio. However, the breakeven ages in Figures 4 and 5 suggest post-age 75 as the optimal time frame to begin evaluation of a possible switch from managing the portfolio to a SPIA. It should be pointed out that the retiree does need to also evaluate their likelihood that they are in the group which may outlive 70% of their cohorts in this example and if the original bequest motive is now a lower priority than outliving their assets.

Annual Payout Rates

As defined by ImmediateAnnuities.com, the Annual Payout Rate (APR) is the percentage of the purchase price (i.e., the premium) which is paid back to you each year and includes both interest and return of principal. The Payout Rate is NOT an interest rate. It is significantly higher than the actual interest rate credited to the premium.

The APR values derived in this paper are the result of a search function, described in practical application below, that solves for an equal value of total cash flow sum for the SPIA with the total cash flow sum for the targeted Monte Carlo simulation value (by allocation and fee level). Thus, a practitioner can compare past, present or future threshold APR values as demonstrated in Figure 6.

Figure 6. SPIA Annual Payout Rate (APR) Table Comparing 50% Equity to 20% Equity

Degree of Sensitivity to Annual Payout Rate (APR)

		Target APR @ 50% Equity and 1% Portfolio Fee			
		With Ending Portfolio Balances		**Without Ending Portfolio Balances**	
	APR	Expected Longevity	Extended Longevity	Expected Longevity	Extended Longevity
Age	2013	50% Outlive	30% Outlive	50% Outlive	30% Outlive
65	6.20%	11.61%	11.28%	7.77%	7.94%
70	6.93%	12.21%	11.59%	8.05%	7.81%
75	8.29%	13.64%	12.19%	8.60%	8.09%
80	10.33%	15.64%	13.61%	9.81%	8.63%
85	13.15%	19.84%	16.84%	11.89%	9.77%
				Target APR is LESS than 2013 APR	
		Target APR @ 20% Equity and 1% Portfolio Fee			
		With Ending Portfolio Balances		**Without Ending Portfolio Balances**	
	APR	Expected Longevity	Extended Longevity	Expected Longevity	Extended Longevity
Age	2013	50% Outlive	30% Outlive	50% Outlive	30% Outlive
65	6.20%	8.89%	8.16%	6.40%	6.19%
70	6.93%	9.82%	8.87%	6.94%	6.44%
75	8.29%	11.51%	9.87%	7.76%	7.01%
80	10.33%	13.71%	11.50%	9.14%	7.80%
85	13.15%	18.02%	14.89%	11.42%	9.21%
				Target APR is LESS than 2013 APR	

The calculated Breakeven APR in Figure 6 targets setting the breakeven point for the annuity line specifically at 50% equity allocation, 1% fee (upper half); or 20% equity allocation with 1% fee (lower half). This is specifically done to see what APR is required as a breakeven comparison. In this manner, past, present and future APRs may be calculated and compared in such a manner to determine what APR may suggest a SPIA or retaining the managed assets.

The basic question a retiree may ask themselves about a managed portfolio is with what allocation are they comfortable with? If they are comfortable with an allocation above the SPIA breakeven line, while also considering the range of possible future market outcomes (between continuous poor markets depicted at 75th percentile, and continuous good markets depicted at 25th percentile, or somewhere in-between) then the retiree should opt to continue to manage their assets outside of a SPIA. In this case, they would also evaluate their situation using the insight provided with data that retains the ending portfolio values. If the retiree is uncomfortable with any allocations above the SPIA breakeven line, then they should consider buying a SPIA with their assets. In which case, the evaluation should use data that does not retain the portfolio balances since those would be given up to buy the SPIA. With this in mind, how does a retiree determine their sensitivity to asset allocation and the SPIA breakeven line through use of the APR as the decision metric?

Those targeted APRs are derived values in order for a retiree, or their adviser, to compare SPIA APRs at any given time to a targeted APR derived from setting a target cash flow breakeven. The yellow highlighted APRs in Figure 6 suggest that if a retiree firmly believes they may outlive their cohorts, then buying a SPIA at any age 65 and older should be considered. Notice though, that if their belief is that they may outlive only 50% of their cohorts, then age 75 or 80 is when they should consider buying a SPIA with their assets, depending on their comfort level with equity exposure (50% and 20% equity shows in Figure 6).

Using the Annuity period life table results in longer distribution periods, due to the "healthier" population represented by the tables' longer longevity. The effect of longer DPs is to lower the withdrawal rate, across all ages, relative to the withdrawal rate of the Social Security (general population) tables. The result is to preserve the portfolio value, at all ages relative to the Social Security table, which extends the distribution ability of the managed portfolio by conserving the portfolio value for later spending.

APR - A retiree can compare the current APR to the threshold APR to determine whether they should opt for the SPIA or retain the managed portfolio until such time as the current APR exceeds the threshold APR as described in Practical Application section later.

General Observations for this latter half of the paper

What the reader may observe is that the higher SPIA payout, that occurs from the start relative to the lower managed portfolio payout at the start, is overcome over time by the greater REAL returns that raise the total payout sum for the managed portfolio and thus the total REAL sum of money the retiree receives is greater for the managed account (to what extent depends on the asset allocation). For older starting ages, where the SPIA payout is, again, higher initially relative to managed, there is not enough time for the combination of inflation (that lowers the real payout of the SPIA) and the effect of real returns (that raises the managed payout – degree depends on allocation again) to overcome the SPIA initial payout advantage.

The Monte Carlo approach in these papers shows the spread between continual poor markets and continual good markets which shows us the comparison of the SPIA between both of those kinds of markets. That spread provides valuable insight into the impact market sequences may have on the decision. A retiree's Transient Stage (JFP Nov 2011 paper describes this) would migrate over time between those percentile depictions rather than stay precisely on the median markets values because of what the market and economic cycles do in the future. What the authors' findings provide is more information about what factors to consider … some retirees may be comfortable with higher equity allocations which would mean they would remain managed longer, while others may be uncomfortable with equity and be inclined to tilt towards a SPIA decision. This decision then needs to consider where their health suggests they may be relative to their cohorts since the healthier, older, retiree who may outlive 70% of their cohorts, should consider a SPIA as a stronger option versus the "normal expected longevity" retiree.

Another important variable is how well funded a client is with regard to their retirement plans. Someone who is very well funded can afford to take greater equity risk and doesn't need to worry about an annuity. Someone who is more constrained cannot afford the equity risk and needs to work more toward building the floor with fixed income. With a higher bond allocation, the expected return will be less and it will be easier for the annuity to provide a material benefit. Social Security (and a pension if the retiree has one) should also be considered as fixed income sources. The breakeven point between just how much equity/bond a retiree may need as evaluated with their comfort level and capacity with that required portfolio risk may be compared with the expected SPIA outcome. How can they make that decision today? The authors' graphs and findings provide insight into what age, at what allocation, and in Phase II, at what Annual Payout Rate (APR) they should favor the SPIA over managed, or vice versa. So, if they need more equity than they may be comfortable with, then the annuity may be more beneficial. However, this is only if the SPIA breakeven point is ABOVE the allocation point at which the retiree would be uncomfortable.

Inflation risk goes down with shorter time frames because the retiree is exposed to the effect of inflation for less time. The challenge is to balance equity allocations to overcome the inflation effect for the given time period, against the fluctuation in portfolio values that a given equity allocation exposes the retiree to. The SPIA payout amount in dollar terms is always higher relative to the portfolio payout rate at any given age. Thus, it takes more time for the portfolio growth rate to overcome this payout advantage; with longer time periods, and higher equity exposure being the rule in general. If retaining value for bequest motives is a goal, clearly using all of the portfolio value for SPIA purchases would not work well.

Tables that have a shorter expected longevity period will favor managed portfolios while tables that have a longer expected longevity period will favor SPIA solutions. This is logical since SPIAs are, by design, not outlived through the use of additional funds first of other not so lucky SPIA retirees, and lastly by insurance company general funds as the distribution period gets longer and longer for those who continue to live. Thus, this advantage only goes to the long lived. Regardless of which table to use, the fundamental observation between both Part 1 and 2 is that the decision to use a SPIA should be delayed to older ages.

Additionally, the methodology used in this research from FMB 2012b utilizes a mathematical adjustment to the withdrawal rate (adjustment = WR% * (1 – 1/n) where n equals the DP from the life table). This adjustment reduces the simulation derived WR, hence the withdrawn dollars for the year, thus giving additional advantage to the SPIA values. This "superannuation" adjustment "pushes" the portfolio balances into future years precisely to retain portfolio values for consumption purposes to offset the retiree's concern of outliving their money.

Comparing the sum of the present values of the annual SPIA payments over the same time period as the sum of the expected annual payments from a portfolio, with sensitivity analysis as to both asset allocation differences in these payments, and consideration of the range between poor market sequence sums and good market sequence sums, a retiree gets deeper insight into factors they should consider when deciding between keeping some or all of their portfolio or purchasing a SPIA.

Practical application

With insights from Part 1, and with an understanding of the effect of using shorter vs. longer life tables, combined with APR as a tool that may be used for measurement, let us look at practically applying APR as a breakeven decision tool.

Set 1: For lifetime expected total cash flow sum from a SPIA.

Step 1.

a. Determine the baseline DP for comparison purposes. Beginning with the retiree's current age (and that of spouse if any), refer to a Period Life Table for the *expected number of years for their longevity* (joint if with a spouse). Note: you may also refine this by using a percentile within the life table to accommodate the retiree's degree of expectation of outliving their cohorts.
b. In excel, make a **column** with each year counting down by one, beginning with the value from Step 1.a., for the total number of years.
c. Note: This step establishes the total time period over which the sum of cash flows may be compared between Set 1 and Set 2.

Step 2.[3]

a. Recall from Part 1 that the expected lifetime sum of payments for the SPIA = $\sum_1^N$(monthly$SPIA*12)/((1+i)^N), where N=number of years of total expected lifetime payments received, and i=inflation rate. Above a second column, enter the monthly SPIA dollar amount the retiree(s) may receive (**monthly$SPIA**). Note: inflation rate used here should be the same inflation rate used in the Monte Carlo Set 2 below.
b. In excel, run the formula in Step 2a for each year (row) in a **second column**.
c. Sum column two, the annual values from step 2b, to arrive at the total expected SPIA lifetime sum. **SPIA Total Expected Cash Flow.**

Set 2: For lifetime expected total cash flow sum from a managed portfolio.

Step 3.

a. For the *first* Monte Carlo simulation period, use the DP from step 1a. Annotate this DP in a third column in the first row of values from Set 1 above.
b. For the *second* Monte Carlo simulation period, age the retiree(s) one year and *refer* to the life tables for the new simulation DP based on these ages. Annotate this DP in the **third column** in the next row of values from Set 1.
c. For the *third* Monte Carlo simulation period, age the retiree(s) yet one more year and *refer* to the life tables based on these ages. Again, annotate in excel.
d. Repeat until you have completed the third column for each row of years from Set 1.
e. Note: The DP periods for Monte Carlo do not change evenly by 1 year at a time because, for each age, there will always be an older expected age in the life tables (at least as far as the tables calculate to).

Step 4. (Generic since Monte Carlo simulators differ).

a. Set up Monte Carlo simulation for DP from Step 3a and retiree's asset allocation and portfolio balance. The cash flow is gross with adviser fees and taxes coming from the gross annual cash flow.
    1. Determine the years' withdraw dollar amount by targeting a percentage of simulations that fail (or that succeed); e.g., 10% (90%).
    2. Annotate the annual gross withdrawn dollar value from 4.a.1. in a **fourth column** for the year's cash flow at present age.
b. Determine the probable portfolio balance at the end of the year (e.g., MoneyGuidePro: set the number of distribution years to 1).

Step 5.[4]

a. Repeat Step 4 using the *next* DP from Step 3b. However, the beginning portfolio balance for this iteration would be the balance from step 4.b.
b. Repeat Step 4 using each sequential DP from Step 3b until all the rows are completed in the fourth column.

c. Note: FMB 2012a and 2012b describe the methodology of annually recalculating the withdrawal and portfolio balances and serially connecting the results. Steps 4 and 5 describe this methodology further. The methodology provides more insight into prudent management of withdrawals since it continually refers back to the portfolio value at any given time (as would occur in real life during annual reviews) and continually updates the DP (as would also occur during annual reviews).
d. Sum the annual values in column four from step 5b to arrive at the total expected Managed Portfolio lifetime sum. **Managed Portfolio Total Expected Cash Flow.**
e. Note: The above does not adjust the WR by 1-1/n, which preserves portfolio balances for use at subsequent older ages, as was done in this research project. The practitioner may apply this adjustment by reducing each year's calculated withdrawal dollar amount (or directly to WR if reader's Monte Carlo simulator allows) in step 4 accordingly.

Set 3: Deriving the APR.
Step 6.

a. In excel, in any new cell, enter formula: **SPIA Total Expected Cash Flow - Managed Portfolio Total Expected Cash Flow.** (Target "Set Cell" cell below).
b. In excel, go to "Data" and select "What if Analysis."
c. Click on "Goal Seek."
    1. In the top box, enter the cell name, e.g., E26, which corresponds to Step 6.a.
    2. In the middle box, enter zero.
    3. The bottom box is the cell, e.g. B1, where you entered the monthly SPIA payment (Monthly$SPIA) value.
    4. Note: Goal Seek is asking: "To what value do I change Monthly$SPIA (SPIA payment) to make the difference between SPIA and Managed Portfolio = zero."

Step 7.

a. In a new cell, multiply the monthly SPIA payment by 12 months and divide is by the retiree's portfolio value. Result is the Breakeven Annual Payout Rate (APR).
b. Compare the quoted APR to the breakeven APR in Step 7a.
    1. If quoted APR is above the current APR, then a SPIA is favorable for the retiree, after considering and discussing the retiree's health outlook, any bequest motives, full or partial use of a SPIA, comfort with asset allocation used in the comparison, etc.
    2. If quoted APR is below the breakeven APR, then a SPIA is currently unfavorable for the retiree and the decision may be delayed for a later age when the comparison may be repeated with new data in the future.

Closing Comments

Serial connection of the simulations reflects real life when one considers how subsequent annual reviews actually occur of a retiree's distribution. The difference is that the series of simulations use the same data set. In real life, each annual review would update the data set which would adjust those updated simulations slightly going forward (Mitchell, 2013).[5] This also has the effect of having the retiree make small adjustments over time, rather than making larger adjustments based on expectations of what might occur that actually don't then occur. The methodology introduced in this paper is not the final answer; the reader should not rely on today's tables and figures. They are meant to illustrate the methodology and decision making that requires a dynamic insight. Rather, using data known in the future and the practical application process described above, future breakeven points would emerge for decision making at that time. These figures and graphs are simply snapshots of moving frames over time.

The practitioner with a good understanding of the factors that affect a breakeven comparison, as discussed in Parts 1 and 2, will be of great value to their retirees.

Abbreviated Terms

APR	Annual Payout Rate
DP	Distribution Period
FMB	Frank, Mitchell, and Blanchett
LP	Longevity Percentiles inherent within Period Life Tables
POF	Probability of Failure (Percentage of simulations that fail the given DP)
WR	Withdrawal Rate

Endnotes:

[1] A working paper, which includes appendices, data and figures, for some calculations in this summary paper, is available at SSRN: http://ssrn.com/abstract=2050003.

[2] California has an annuitized annuity premium tax of 2.35%/0.5% non-qualified/qualified. Five other states have annuitized premium taxes between 1.0% and 3.5% (Virgin Islands 5.0%). This tax effect lowers the total sum of the SPIA value by that amount. This tax effect does not have a significant impact on results. For example, a Texas (no premium tax) 60-year old female with $1,000,000 is quoted $4,586/month vs. California $4,479. For an expected longevity of 26 years the Texas total nominal sum equals $1,430,832 vs. $1,397,388 in California. The difference of $33,384 is the long term effect of the tax.

[3] An advanced excel user could program excel to do the simulations as well as track and sum year end values for Set 2.

[4] After any given period, an even older longevity for those still alive always exists. Thus an additional distribution period needs to be planned by extending the distribution period as longevity age extends by that aging (FMB 2012a&b). Rather than depend on a single simulation, the methodology more closely reflects real life by annually revisiting key factors, updating them, and serially connecting the results of a series of simulations. Using this approach, the managed portfolio not only manages for income in the present year, but also manages the portfolio balance for potential income in future years; an eye on both income and balances results, which provides more insight into total lifetime income than any single simulation alone.

[5] Updating, over time in the future, stochastic data that is variable and uncertain in nature (e.g., market returns, SPIA rates, longevity, etc.) allows risks that may be amplified by long periods of time, to be less amplified when less time remains in the future. In other words, the dispersion of possible results is less for shorter periods of time relative to longer periods.

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